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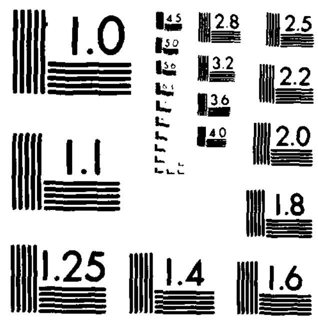

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ADAPTIVE ARMA SPECTRAL ESTIMATION

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ABSTRACT

A novel adaptive method for efficiently obtaining an ARMA model spectral estimate of a wide-sense stationary time series is presented. It is adaptive in the sense that as a new element of the time series is observed, the coefficients of a (p,p)th order ARMA model may be algorithmically updated. This algorithm's computational complexity (i.e., the number of multiplications and additions required) is of the order p log(p) for a particular version of the method. Moreover, the spectral estimation performance of this new method is found typically to be far superior to such contemporary approaches as the Box-Jenkins, maximum entropy, and, Widrow's LMS methods. This performance in conjunction with its computational efficiency mark this algorithm as being a primary spectral estimation tool.

I. *INTRODUCTION*

In various signal processing applications, it is necessary to estimate the power spectral density of a wide-sense stationary time series $\{x_n\}$. Since only a finite set of time series' observations are typically available for this task, one almost always invokes a finite parameter model for the spectral density estimate. Without doubt, the rational spectral density function as specified by

$$S_x(\omega) = \frac{|b_0 + b_1 e^{-j\omega} + \dots + b_q e^{-jq\omega}|^2}{|1 + a_1 e^{-j\omega} + \dots + a_p e^{-jp\omega}|^2}\sigma^2 \qquad (1)$$

constitutes the most widely used of such models. This particular model is generally referred to as being an autoregressive-moving average (ARMA) model of order (p,q).

The predominant effort in rational spectral estimation has been directed towards the more specialized autoregressive (AR) model for which q = 0, and, the moving-average (MA) model in which p = 0. As examples, the maximum entropy, one-step predictor, and autoregressive methods have been developed for efficiently estimating the AR model's a_k coefficients. Similarly, the periodogram and its variants have been found to yield effective MA modeling procedures. The interested reader will find excellent treatments of these and other rational spectral estimation methods in Haykin [1] and Childers [2].

Recently, attention has been focused on developing so-called super-efficient algorithms for estimating the AR model's a_k coefficients whereby on the order of p log(p) computations are required for this task. These algorithms are typically predicated on the divide and conquer approach (e.g., see refs. [3]&[4]). These super algorithms offer the potential of providing a significant computational advantage in generating AR spectral estimates when compared to other contemporary AR procedures. Unfortunately, implementation of these super algorithms is relatively complex and a rather large value for the AR order parameter p is required before the computational complexity p log(p) is approached. It is felt that future developments will alleviate these difficulties.

Despite the concentrated interest given to AR spectral estimation, it is widely recognized that an ARMA spectral model is generally the most effective rational model from a parameter parsimony viewpoint. In recognition of this fact, a variety of procedures have been developed for generating ARMA models. These include the whitening filter approach which is typically iterative in nature, generally slow in convergence, and, usually requires an excessively large number of time series' observations to be effective (e.g., see refs. [5] & [6]). More desirable closed form procedures which overcome these deficiencies have been offered. These include the so-called Box-Jenkins method and its variants [7] - [9], and, more recently, Cadzow has developed a "high performance" method [10] & [11]. Although this latter method has provided excellent spectral estimation performance when compared to the maximum entropy and Box-Jenkins methods, its computational efficiency is somewhat inferior.

We shall herein present a novel algebraic approach for generating an ARMA model spectral estimate. It offers the dual advantage of having a super algorithm's computational efficiency while at the same time maintaining a spectral estimation

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capability similar to the above mentioned high performance method. These characteristics mark this algorithm as being a primary spectral estimation tool.

In this paper, we shall consider exclusively the task of estimating the ARMA model's autoregressive coefficients. This estimation is, to a large extent, motivated by the well-known Yule-Walker equations which will be briefly reviewed in the next section. Once these autoregressive coefficient estimates have been obtained, a variety of procedures exist for estimating the ARMA model's moving-average coefficients. The interested reader may consult references [10] & [11] for a description of such procedures.

II. FUNDAMENTAL CONCEPTS

It is readily shown that the random time series with spectral density (1) can be modeled as being the response of the causal ARMA system

$$x_k + \sum_{i=1}^{p} a_i x_{k-i} = \sum_{i=0}^{p} b_i \varepsilon_{k-i} \qquad (2)$$

to the zero mean white noise excitation $\{\varepsilon_n\}$ whose individual terms have variance σ^2. Although the more general case may be straightforwardly treated, we have here restricted $q = p$ for purposes of ease of understanding. The autocorrelation characterization of this ARMA system is readily achieved by first multiplying each side of equation (2) by the entity x^*_{n-m} and then taking the expected value. This results in the well known Yule-Walker equations as given by

$$r_x(m) + \sum_{k=1}^{p} a_k r_x(m-k) = 0 \qquad \text{for } m > p \qquad (3)$$

In this equation, the symbol $r_x(m)$ denotes the time series' autocorrelation sequence

$$r_x(m) = E\{x_n x^*_{n-m}\} \qquad (4)$$

where $*$ and E denote the operations of complex conjugation and expected value, respectively.

In what is to follow, the Yule-Walker equations (3) will serve as a motivating influence in evolving a method for estimating the autoregressive coefficients of the ARMA model (2). These autoregressive coefficient estimates are to be based totally on the following contiguous set of n time series observations

$$x_1, x_2, \ldots, x_n \qquad (5)$$

The method to be described will be adaptive in nature. Namely, as the new time series element x_{n+1} becomes available, it is possible to efficiently update the optimal autoregressive coefficients generated from the n data set (5) to obtain the optimal autoregressive coefficients corresponding to the enlarged $n+1$ data set (i.e., $x_1, x_2, \ldots, x_{n+1}$).

III. ARMA ESTIMATION: DIRECT APPROACH

In this section, a procedure for estimating the ARMA models' autoregressive coefficients shall be given. To begin this development, one first evaluates the model equation (2) over the set $p+2 \le k \le n$ to obtain the following time series' relationships

$$\begin{bmatrix} x_{p+2} \\ x_{p+3} \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} x_{p+1} & x_p & \cdots & x_2 \\ x_{p+2} & x_{p+1} & \cdots & x_3 \\ \vdots & \vdots & & \vdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-p} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} \varepsilon_{p+2} & \varepsilon_{p+1} & \cdots & \varepsilon_2 \\ \varepsilon_{p+3} & \varepsilon_{p+2} & \cdots & \varepsilon_3 \\ \vdots & \vdots & & \vdots \\ \varepsilon_n & \varepsilon_{n-1} & \cdots & \varepsilon_{n-p} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_p \end{bmatrix} \qquad (6a)$$

It will be convenient to represent this relationship in the more compact vector format

$$\underline{x} + X\underline{a} = \mathcal{E}\,\underline{b} \qquad (6b)$$

where $\underline{x}$, $\underline{a}$, and $\underline{b}$ are $n-p-1$, p, and, $p+1$ column vectors respectively, while X and $\mathcal{E}$ are $(n-p-1) \times p$ and $(n-p-1) \times (p+1)$ matrices, respectively. The entries of these vectors and matrices are obtained by directly comparing relationships (6a) and (6b).

We now wish to use the Yule-Walker equations (3) in conjunction with relationship (6) to estimate the autoregressive coefficient vector $\underline{a}$. This objective is readily achieved by first introducing the following $(n-p-1) \times t$ lower triangular type matrix

$$Y = \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ x_2 & x_1 & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \vdots & \vdots & & x_1 \\ \vdots & \vdots & & \vdots \\ x_{n-p-1} & x_{n-p-2} & \cdots & x_{n-p-t} \end{bmatrix} \qquad (7)$$

The selection of the integer t, which specifies the number of columns of matrix Y, is critical. A discussion of how one goes about making this selection will be shortly given.

Upon left multiplying each side of relationship (6b) by the complex conjugate transpose of matrix (7) as denoted by Y^{+}, there results

$$Y^{+}\underline{x} + Y^{+}X\underline{a} = Y^{+}\mathcal{E}\,\underline{b} \tag{8}$$

This system of equations is readily found to constitute a statistical approximation to the first t Yule-Walker equations (i.e., expression (3) for $p < m \le p+t$). This is readily verified by taking the expected value of expression (8) which results in the following set of equations

$$(n-m)\left[r_x(m) + \sum_{k=1}^{p} a_k r_x(m-k)\right] = 0 \quad \text{for} \quad p < m \le p+t \tag{9}$$

It is to be noted that the right side zero term arises due to the fact that $E\{Y^{+}\mathcal{E}\} = \Theta$ (i.e., the null matrix). This is a direct consequence of the ARMA models' causality and the whiteness of the excitation time series which causes $E\{x_n^{*}\varepsilon_k\} = 0$ for $k > n$.

With the above observations in mind, a logical selection procedure for the autoregressive coefficient vector is suggested. Namely, the autoregressive coefficient vector $\underline{a}$ is selected so as to cause the left side of relationship (8) to be as close as possible to the zero vector (i.e., the expected value of the right side vector $Y^{+}\mathcal{E}\,\underline{b}$). This results in an approximation to the Yule-Walker equations which is "most" consistent with the time series' observations (5). A particularly convenient measure of the closeness of $Y^{+}\mathcal{E}\,\underline{b}$ to the zero vector is given by the quadratic functional

$$f(\underline{a}) = (Y^{+}\underline{x} + Y^{+}X\underline{a})^{\dagger}W(Y^{+}\underline{x} + Y^{+}X\underline{a}) \tag{10}$$

in which W is a $t \times t$ symmetric positive semi-definite matrix that is usually selected so as to weight differently various elements of the error vector $Y^{+}\underline{x} + Y^{+}X\underline{a}$. It is readily shown that the autoregressive coefficient vector which minimizes this quadratic functional satisfies the following consistent system of p linear equations in the p autoregressive coefficient unknowns

$$X^{+}YWY^{+}X\underline{a}^{\circ} = -X^{+}YWY^{+}\underline{x} \tag{11}$$

It is possible to use projection theory concepts to achieve a computationally efficient method for obtaining the optimum vector $\underline{a}^{\circ}$ when the weighting matrix W is exponentially diagonal. A paper now in preparation will detail this solution procedure.

A few words are now appropriate concerning the selection of the integer t which in part characterizes matrix Y as given by expression (7). If t is set equal to p, it is seen that relationship (7) constitutes a statistical estimate of the first p Yule-Walker equations. As such, relationship (11) bears a resemblance to the Box-Jenkins method of autoregressive coefficient estimation [7]. Upon closer examination, however, it is found that these two approaches are quite different. As a matter of fact, it has been empirically found that the spectral estimation performance which results from utilization of relationship (11) with t = p is distinctly better than that obtained with the Box-Jenkins method. [11].

If the integer t is taken to be larger than p, then more than the minimal number (i.e., p) of Yule-Walker equation approximations are generated. With this larger base of Yule-Walker equation approximations (i.e., greater than p), it might be conjectured that an improvement in spectral estimation performance would result. This anticipated improvement in performance has been in fact empirically demonstrated on numerous examples treated to date.

It is readily shown that the procedure here presented for selecting the autoregressive coefficients is equivalent to Cadzow's high performance ARMA spectral estimation method [10]&[11]. As such, a large base of empirical evidence gathered in using this latter procedure suggests that the spectral estimation performance of this paper's procedure is clearly superior to that achieved by such commonly used contemporary procedures as the Box-Jenkins and maximum entropy methods. The advantage accrued in algebraically formulating the spectral estimation problem via relationship (8) (as compared to the equivalent high performance method) resides in the ability to directly use sophisticated least mean square concepts. Using these concepts, it is possible to evolve a computationally efficient adaptive spectral estimation algorithm whose computational complexity is on the order of p^2. More importantly, by an appropriate modification of the vector $\underline{x}$ and matrices X and Y used in relationship (8), it is possible to obtain a truly super computational algorithmic procedure which has a computational complexity of the order p log(p). It is this capability which distinguishes the herein presented method from the high performance method.

IV. ARMA ESTIMATION: MODIFIED APPROACH

It is possible to realize a significant computational improvement in the proposed spectral estimation algorithm by restricting t to be p and by appropriately modifying the vector and matrix entrees in expression (8). Although a variety of such modifications are feasible, we will here restrict our interests to two such possibilities called the "premodification" and "postmodification" approaches. Additional modifications are being examined and will be shortly reported upon.

(a) Premodification Method

In the premodification method, the $\underline{x}$ vector and Y matrix as given in expressions (6) and (7) remain the same while the integer t is fixed at p. The matrix X, however, is modified to the following lower triangular format

$$X = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ x_{p+2} & 0 & 0 & \cdots & 0 \\ x_{p+3} & x_{p+2} & 0 & \cdots & 0 \\ \vdots & \vdots & x_{p+2} & \ddots & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ \vdots & \vdots & & & x_{p+2} \\ \vdots & \vdots & & & \vdots \\ x_{n-1} & x_{n-2} & \cdots & \cdots & x_{n-p} \end{bmatrix} \tag{12}$$

If this matrix is substituted into relationship (8), an alternate method for estimating the autoregressive coefficients is at hand. Due to the lower triangular structure of both matrices X and Y, however, it is possible to implement an adaptive algorithm for obtaining such estimates with a computational complexity of order p. This remarkable improvement in computational efficiency is found to be dependent on a lattice structure implementation of the ARMA model (e.g., see ref. [13]).

A measure of the spectral performance of this premodification method may be obtained by taking the expected value of expression (8) with the matrix substitution. This is readily found to yield

$$(n-m)\sum_{k=0}^{m-p-1} a_k r_x(m-k) + \sum_{k=m-p}^{p} (n-p-k-1)a_k r_x(m-k) = 0 \qquad p < m \leq 2p \tag{13}$$

where the a_0 coefficient is set equal to one. Although these relationships don't precisely satisfy the Yule-Walker equations as in the *direct* approach, it is noted that for $n \gg p$, an excellent approximation is in fact realized. With this in mind, it is reasonable to anticipate that this premodification method will have a spectral estimation performance approaching that of the aforementioned high performance (direct) method. Empirical evidence gathered to date re-enforces this conjecture.

b Postmodification Method

In the postmodification method, the alterations to be made are given by

$$\underline{y}' = [x_{p+2} \quad x_{p+3}, \ldots, x_n, 0, 0, \ldots 0] \tag{14a}$$

$$Y' = \begin{bmatrix} x_{p+1} & x_{p+2} & \cdots & x_n, 0, & \cdots & 0 \\ x_p & x_{p+1} & \cdots & x_{n-1} \quad x_n & 0, & \ldots 0 \\ \vdots & \vdots & & & \ddots & \vdots \\ \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & & & & 0 \\ x_2 & x_3 & \cdots & \cdots & \cdots & x_n \end{bmatrix} \tag{14b}$$

$$Y'' = \begin{bmatrix} x_1 & x_2 & \cdots & \cdots & x_{n-1} \\ 0 & x_1 & \cdots & \cdots & x_{n-2} \\ \vdots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0, & x_1, \ldots, & x_{n-p} \end{bmatrix} \tag{14c}$$

where the prime denotes the transpose operator. Using these entries in expression (8), it is possible to evolve an adaptive algorithmic solution procedure for the optimum autoregressive coefficients. This entails utilization of the doubling algorithm concept and results in an adaptive algorithm whose computational complexity is on the order of p log(p) [13].

To gauge the effectiveness of this postmodification method as embodied in expression (8), the expected value of this expression with entries (14) is next taken and results in

$$(n-m)r_x(m) + \sum_{k=1}^{p} (n-m+k)a_k r_x(m-k) = 0 \qquad p < m \leq 2p \tag{15}$$

As in the premodification method, these expected value relationships do not precisely satisfy the governing Yule-Walker equations. On the other hand, for $n \gg p$, it is clear that they do provide a very excellent approximation to these characteristic equations. As such, it is not surprising that the autoregressive coefficient estimates provided by the postmodification method as represented by expression (8) yield a very satisfactory spectral estimation performance. This performance has been empirically found to approach that of the high performance (direct) method.

In order to test the effectiveness of the herein proposed ARMA spectral estimation approach, the classical problem of resolving two sinusoids embedded in additive white noise was considered. The spectral estimates obtained from these methods are shown in Fig. 1 along with the results generated using the maximum entropy and Box-Jenkins methods. It is apparent that the estimates obtained using this paper's approach were clearly superior for this task.

V. CONCLUSION

A novel approach to ARMA model spectral estimation has been presented. This estimation approach possesses the dual attributes of providing an excellent spectral estimation performance, and, of having an exceptional computational efficiency. Its excellent spectral estimation performance has been demonstrated on numerous examples treated to date and virtually always exceeds that achieved by such contemporary procedures as the maximum entropy and Box-Jenkins Methods.

The above mentioned super computational efficiency was achieved by appropriately modifying some vector and matrix entries. In particular, two such modifications have been herein offered. Further studies are now being conducted relative to employing the basic approach herein taken to evolve even better performance. These results will be reported in forthcoming papers.

VI. REFERENCES

[1] S. Haykin (Editor) NONLINEAR METHODS OF SPECTRAL ANALYSIS, Springer-Verlag, New York, 1979.

[2] D. G. Childers (Editor), MODERN SPECTRAL ANALYSIS, IEEE Press, New York, 1978.

[3] M. Morf, "Doubling Algorithms for Toeplitz and Related Equations" in Proc. 1980 IEEE Int. Conf. Acoust., Speech, Signal Processing, Denver, CO, Apr. 9-11, 1980, pp 954-959.

[4] Robert R. Bitmead and Brian D. O. Anderson, "Asymptotically Fast Solution of Toeplitz and Related Systems of Linear Equations" Technical Report No. EE7915.

[5] S. A. Tretter and K. Steiglitz, "Power Spectrum identification in terms of rational models", IEEE Trans. Automat. Contr., vol. AC-12, pp 185-188, Apr., 1967.

[6] P. R. Gutowski, E. A. Robinson, and S. Treitel, "Spectral estimation, fact or fiction", IEEE Trans. Geosci. Electron., vol. GE-16, pp 80-84, Apr., 1978.

[7] G. Box and G. Jenkins, TIME SERIES ANALYSIS: FORECASTING AND CONTROL (revised edition). San Francisco: Holden-Day, 1976.

[8] M. Kaveh, "High resolution spectral estimation for noisy signals", IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-27, pp 286-287, June, 1979.

[9] J. F. Kinkel, J. Perl, L. Scharf, and A. Stubberud, "A note on covariance-invariant digital filter design and autoregressive moving average spectrum analysis", IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-27, pp 200-202, Apr., 1979.

[10] J. A. Cadzow, "ARMA spectral estimation: A model equation error procedure", IEEE Trans. on Geoscience and Remote Sensing, vol. GE-19, No. 1, pp. 24-28, Jan. 1981.

[11] J. A. Cadzow, "High Performance Spectral Estimation - A New ARMA Method", IEEE Trans. on Acoust., Speech, Signal Processing, vol. ASSP-28, No. 5, pp 524-529, Oct., 1980.

[12] M. Morf and D. T. Lee, "Recursive Least Square Ladder Forms for Fast Parameter Tracking", Proc. 1978 IEEE Conf. D & C, San Diego, CA, pp 1362-1367, Jan. 12, 1978.

[13] J. A. Cadzow and K. Ogino, "Adaptive rational spectral estimation", paper in preparation.

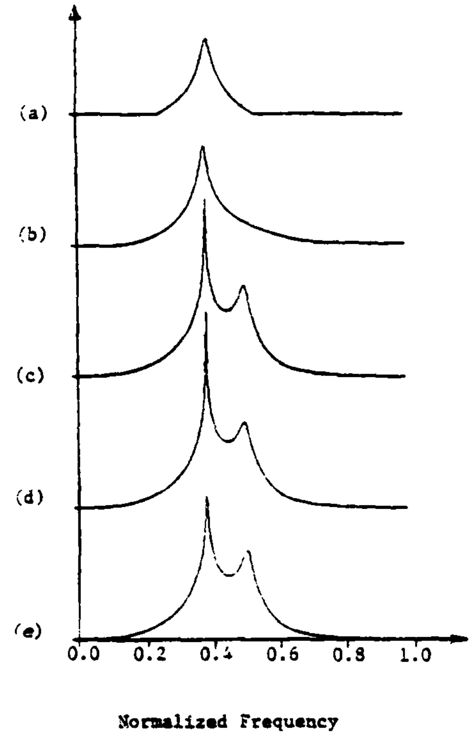

Fig. 1. Spectrum estimates of the time series

$$x_n = \sqrt{20}\cos(0.4\pi n) + \sqrt{2}\cos(0.5\pi n) + \omega(n) \qquad 0 \le n \le 159$$

consisting of two sinusoids at frequencies 0.4 (10dB) and 0.5 (0dB) embedded in additive white noise of variance one: (a) Fourth Order AR Model using the maximum entropy covariance method, and, ARMA models of order (4,4) using the (b) Box-Jenkins Method, (c) Premodification Method, (d) Direct Method, and, (e) Postmodification Method.

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